Algebras for non-uniform topic assignments

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SLALM XXI

Roadmap

Light background

Two lines of research

1. An algebraic semantics for Parry's logic of Analytic Containment ([Fine, 1986]). This has been refined and retooled for logics like **R** and variations of **PAI** (e.g., [Tedder, 2026] and [Ferguson, 2023]).
2. Non-uniform substitutions and logics which are closed under them (e.g., [Logan and Worley, 2025], [Ferguson and Logan, 2025], and [Standefer et al., 2025]).

A worry and a way around it

- Several attempts have been made to pinpoint the largest sublogic of classical logic which is closed under some pet non-uniform substitution. This logic is *not* a relevant one. [Logan, 2026]
- Despite this, the semantics proposed here uses the same intension labels as [Logan and Worley, 2025] and is for a relevant logic.

Roadmap

Logic

- Language $\mathcal{L}$ consisting of *formulas* generated from atomic sentences $p, q, \ldots$, a constant $\mathbf{t}$, and connectives $\neg$, $\rightarrow$, $\wedge$, $\vee$, and $\circ$ as follows.

$$A := p \mid \mathbf{t} \mid \neg A \mid A \rightarrow A \mid A \wedge A \mid A \vee A \mid A \circ A$$

- *Bunches* $\mathcal{B}$ generated from formulas:

$$X := A \mid X, X \mid X; X$$

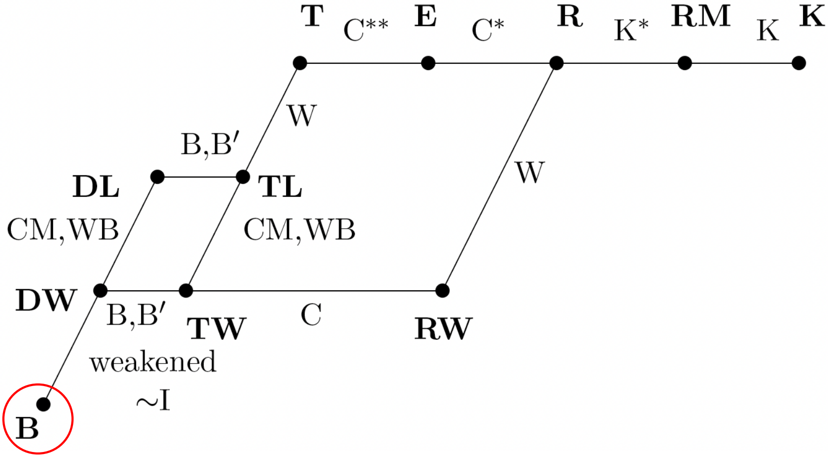
- *Consecutions* $X \succ A$, where $X \in \mathcal{B}$ and $A \in \mathcal{L}$.

Validity

We say $X \vdash A$ whenever there is a proof in $\mathbf{B}_\vee$ that ends in $X \succ A$. For other logics L, we say $X \vdash_{\mathsf{L}} A$ if there is a proof in L that ends in $X \succ A$.

The target logic **B**

- The logic I will be targeting is the relevant logic **B**.
- In particular, a bunched natural deduction presentation for **B**.



λeρicones

We apply a sequence generated from $\{l, r, \lambda, \rho, n\}$ to formulas systematically. ε is short for the empty sequence, and the set of all such sequences is labelled **seq**.

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- The sequence assigned to X^X, where $X \in \mathcal{B}$ is ε.
- If B^A is assigned $\overline{x}$, then

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  - $(B \circ C)^A$ is assigned $\overline{x}\lambda$ and $(C \circ B)^A$ is assigned $\overline{x}\rho$
  - etc.

Roadmap

B algebras

What would it take to interpret **B** algebras ([Meyer and Routley, 1972]) as topic spaces?

$$\langle T, \Box, \oplus, \otimes, \multimap, \downarrow, \geqslant \rangle$$

- **B** algebras are DeMorgan groupoids, with formulas as points.
- They contain a basepoint $\Box$ that sits below the theorems of the logic.
- $A \to B$ is designated iff $A \geqslant B$.

Assignments

Definition 2 (Topic assignment)

Given a topic algebra $\langle T, \Box, \oplus, \otimes, \multimap, \downarrow, \geqslant \rangle$, a *topic assignment* is a function $\gamma : \mathrm{At} \to T$ that extends over $\mathcal{B}$.

Examples:

- $\gamma(A \to B) = \gamma(A) \multimap \gamma(B)$
- $\gamma(A \wedge B) = \gamma(A) \oplus \gamma(B)$

Roadmap

Structure

Definition 3

A *basic topic algebra* is a tuple $\langle T, \square, +, \succeq \rangle$ closed under axioms (t1)-(t3).

(t1) $+ : T \times T \rightharpoonup T$ is a partial function

(t2) $\succeq$ is a relation on $T \times T$.

(t3) $\square \in T$, $\square + \square = \square$, and $\square \succeq \square$

Aboutness

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(v3) $\big(\sigma^{\overline{x}}(p) + \sigma^{\overline{x}}(q)\big) + \sigma^{\overline{x}}(s) = \sigma^{\overline{x}}(p) + \big(\sigma^{\overline{x}}(q) + \sigma^{\overline{x}}(s)\big)$

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(v6) $\sigma^{\lambda}(p) + \sigma^{\rho}(q) \succeq \sigma^{\varepsilon}(s)$ iff $\sigma^{\varepsilon}(p) \succeq \sigma^{l}(q) + \sigma^{r}(s)$

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(v7) $\Box + \sigma^{\rho}(p) = \sigma^{\varepsilon}(p)$

Aboutness redux

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Given a basic topic algebra T, a *varying topic assignment* is a function $\sigma : \mathrm{At} \times \mathbf{seq} \to T$ subject to the following rules.

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i. $\sigma^{\overline{x}}(p) + \sigma^{\overline{x}}(s) \succeq \sigma^{\overline{x}}(q) + \sigma^{\overline{x}}(s)$

ii. $\sigma^{\overline{x}\lambda}(p) + \sigma^{\overline{z}\rho}(s) \succeq \sigma^{\overline{x}\lambda}(q) + \sigma^{\overline{z}\rho}(s)$

iii. $\sigma^{\overline{z}\lambda}(s) + \sigma^{\overline{x}\rho}(p) \succeq \sigma^{\overline{z}\lambda}(s) + \sigma^{\overline{x}\rho}(q)$

iv. $\sigma^{\overline{z}l}(s) + \sigma^{\overline{x}r}(p) \succeq \sigma^{\overline{z}l}(s) + \sigma^{\overline{x}r}(q)$

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Extensions

Extending σ over the language

A varying topic assignment σ extends to a function (also labelled σ) over $\mathcal{B} \times \mathbf{seq}$.

- $\sigma^{\overline{x}}(\mathbf{t}) = \square$
- $\sigma^{\overline{x}}(\neg A) = \sigma^{n\overline{x}}(A)$
- $\sigma^{\overline{x}}(A \rightarrow B) = \sigma^{l\overline{x}}(A) + \sigma^{r\overline{x}}(B)$
- $\sigma^{\overline{x}}(A \wedge B) = \sigma^{\overline{x}}(A) + \sigma^{\overline{x}}(B)$
- $\sigma^{\overline{x}}(A \circ B) = \sigma^{\lambda\overline{x}}(A) + \sigma^{\rho\overline{x}}(B)$

Semantics

A *transparent model*

$$M = \langle T, \Box, +, \succeq, v_M, \sigma_M \rangle$$

is a basic topic algebra with a valuation and a varying topic assignment.

Verification

A transparent model M verifies a consecution $X \succ A$, written $X \Vdash_M A$ iff

(i) $v_M(X) \leq v_M(A)$

(ii) $\sigma_M(X) \succeq \sigma_M(A)$

We write $X \Vdash A$ iff, for every transparent model M, $X \Vdash_M A$.

Results for $\Vdash$

Theorem 6 (Soundness and completeness of $\Vdash$ for $\mathbf{B}_\vee$)

$X \nvdash A$ iff $X \nVdash A$.

Canonical model

Partial canonical model construction

Since $X \nvdash A$, also $X \nvDash A$. Grab the canonical model of the **B**-algebra semantics to build a new canonical model $\langle T, \Box, +, \succeq, v_{\mathsf{CM}}, \sigma_{\mathsf{CM}} \rangle$.

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- $\Box = (\llbracket \mathbf{t} \rrbracket, \varepsilon)$
- $(\llbracket X \rrbracket, \overline{x}) + (\llbracket Y \rrbracket, \overline{y}) = \begin{cases} (\llbracket X \rrbracket \oplus \llbracket Y \rrbracket, \overline{x}) & \overline{x} = \overline{y} \\ (\llbracket X \rrbracket \multimap \llbracket Y \rrbracket, \overline{z}) & \overline{x} = \overline{z}l \text{ and } \overline{y} = \overline{z}r \\ \ldots \end{cases}$

Example

$$X := \langle \text{The man ate the dog} \rangle$$
$$Y := \langle \text{The man went to jail} \rangle$$

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Uniform topic assignment	x	$x \multimap y$	$y \multimap x$

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Sentence	X	$X \to Y$	$Y \to X$
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Non-uniform topic assignment	(x, ε)	$(x, l) + (y, r)$	$(y, l) + (x, r)$

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$$(\llbracket X \rrbracket, \varepsilon) + (\llbracket Y \rrbracket, \varepsilon) = (\llbracket X \wedge Y \rrbracket, \varepsilon)$$
$$(\llbracket X \rrbracket, l) + (\llbracket Y \rrbracket, \varepsilon) = \text{undefined}$$
$$(\llbracket X \rrbracket, \varepsilon) + (\llbracket Y \rrbracket, r) = \text{undefined}$$
$$(\llbracket X \rrbracket, l) + (\llbracket Y \rrbracket, r) = (\llbracket X \to Y \rrbracket, \varepsilon)$$
$$\vdots$$

Is *this* a good theory of topics?

- We have a semantics for $\mathbf{B}_{\vee}$ using structurally underspecified algebras.
- Topics don't generate by randomly combining them.
- The topic of a sentence depends on its logical location.
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- Not every formulable sentence has a topic.
- What about disjunction?

Roadmap

What about all those pesky vels?

- Naive (uniform) approach:

$$\gamma(A \vee B) = \gamma(A) \oplus \gamma(B) \geqslant \gamma(A)$$

- But in **B** algebras,

$$\gamma(A) \geqslant \gamma(A \vee B) = \gamma(A) \ominus \gamma(B)$$

- Is there any way we can seriously defend the notion that the topic of A contains the topic of $A \vee B$ for any B whatsoever?

Conjunction is less than the sum of its parts

The topic of disjunctions

$a \ominus b = \downarrow(\downarrow a \oplus \downarrow b)$

- If we take seriously that $\ominus$ is topic transformative, this means one of $\downarrow$ and $\oplus$ are intensional, topic-transformative operators.
- We cannot interpret $\downarrow a$ as a complement topic of a, since doing so would intimate a principle we do not have in the logic, in particular $\mathbf{t} \vdash A \vee \neg A$
- We could just have easily taken $\ominus$ as the primitive and defined $\oplus$ in terms of it and $\downarrow$. Why privilege one as topic transparent and not the other? The choice is seemingly arbitrary.

Topic fusion is not merely extensional; the fusion of two topics is something distinct from the union of their contents.

A broader class of non-uniform assignments

Adopt a new sequence letter c to track instances of conjunction.

New rules

- For varying topic assignments
  - $\sigma^{\overline{x}c}(p) + \sigma^{\overline{x}c}(q) \succeq \sigma^{\overline{x}}(p)$
  - If $\sigma^{\overline{x}}(p) \succeq \sigma^{\overline{x}}(q)$, then $\sigma^{\overline{x}c}(p) + \sigma^{\overline{x}c}(s) \succeq \sigma^{\overline{x}c}(q) + \sigma^{\overline{x}c}(s)$
- For varying topic assignments over $\mathcal{B}$
  - $\sigma^{\overline{x}}(A \wedge B) = \sigma^{c\overline{x}}(A) + \sigma^{c\overline{x}}(B)$
  - $\sigma^{\overline{x}}(A \vee B) = \sigma^{ncn\overline{x}}(A) + \sigma^{ncn\overline{x}}(B)$

Final updated result

Theorem 7 (Soundness and completeness of $\Vdash$ for $\mathbf{B}$)

$X \vdash_{\mathbf{B}} A$ iff $X \Vdash A$.

Canonical model changes:

$$
\begin{aligned}
&(\llbracket X \rrbracket, \overline{x}) + (\llbracket Y \rrbracket, \overline{y}) \\
&\qquad = \begin{cases} (\llbracket X \rrbracket \oplus \llbracket Y \rrbracket, \overline{z}) & \overline{x} = \overline{y} = \overline{z}c \\ (\downarrow(\downarrow\llbracket X \rrbracket \oplus \downarrow\llbracket Y \rrbracket), \overline{z}) & \overline{x} = \overline{z}ncn \text{ and } \overline{y} = \overline{z}ncn \end{cases}
\end{aligned}
$$

Roadmap

Conclusion

- **B** algebras seem structurally unsuitable to be spaces of topics.
- They uniformly assign topics to formulas and leave it to the logical operators to transform them appropriately.
- The semantic models provided here move the burden of transformation to the non-uniform assignments. As a result, the space of topics explodes but our ability to form new topics is limited by the contextual relevance to each other.
- Disjunction can be given a topic-preserving formalization. Whether this is supported by linguistic data is open for debate.

THANK YOU!

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