Discrete duality for DLH-algebras

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Goal

To present a representation theorem and a discrete duality for Distributive Hilbert algebras.

Contents

Classical correspondences

Logic	**Algebraic semantic**
Intuitionistic logic $\{\vee, \wedge, \rightarrow, 0, 1\}$	Heyting algebra $\langle A, \vee, \wedge, \rightarrow, 0, 1\rangle$

Classical correspondences

Logic	**Algebraic semantic**
Intuitionistic logic $\{\vee, \wedge, \to, 0, 1\}$	Heyting algebra $\langle A, \vee, \wedge, \to, 0, 1\rangle$
Implicative Fragment $\{\to, 1\}$	Hilbert algebra $\langle A, \to, 1\rangle$

Definition (Hilbert algebra)

A Hilbert algebra is an algebra $\langle A, \to, 1\rangle$ where A is a nonempty set, $\to$ is a binary operation on A, 1 is an element of A such that the following conditions are satisfied for every $a, b, c \in A$:

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H2) $(a \to (b \to c)) \to ((a \to b) \to (a \to c)) = 1$

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H1) $a \to (b \to a) = 1$

H2) $(a \to (b \to c)) \to ((a \to b) \to (a \to c)) = 1$

H3) If $a \to b = 1$ and $b \to a = 1$, then $a = b$.

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H2) $(a \to (b \to c)) \to ((a \to b) \to (a \to c)) = 1$

H3) If $a \to b = 1$ and $b \to a = 1$, then $a = b$.

Diego, A. (1961). Sobre álgebras de Hilbert (Doctoral dissertation, Universidad de Buenos Aires. Facultad de Ciencias Exactas y Naturales).

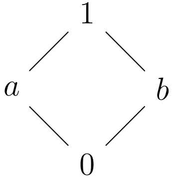
Remark

Poset $\dashrightarrow$ **Hilbert algebra**

$$(A, \leq, 1) \dashrightarrow a \to_{\leq} b = \begin{cases} 1 \text{ iff } a \leq b \\ b \text{ c.c} \end{cases}$$

Two Hilbert implications on the same poset

$$\langle A, \vee, \wedge, 0, 1 \rangle$$



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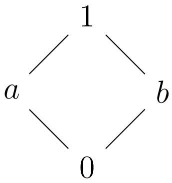

$\to_{\leq}$	a	b	1	0
0	1	1	1	1
a	1	b	1	0
b	a	1	1	0
1	a	b	1	0

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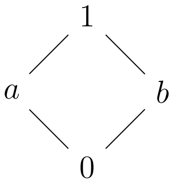

$$a \Rightarrow b = \neg a \vee b$$

$\to_{\leq}$	a	b	1	0
0	1	1	1	1
a	1	b	1	0
b	a	1	1	0
1	a	b	1	0

$\Rightarrow$	a	b	1	0
0	1	1	1	1
a	1	b	1	b
b	a	1	1	a
1	a	b	1	0

Remark

Poset		**Hilbert algebra**
$(A, \leq_{\rightarrow}, 1)$	$\dashleftarrow$	$\langle A, \rightarrow, 1\rangle$
$a \leq_{\rightarrow} b$ iff $a \rightarrow b = 1$		

Different Hilbert implications determine the same poset

Different Hilbert implications determine the same poset $\langle A, \to_{\leq}, 1\rangle$

$\to_{\leq}$	a	b	1	0
0	1	1	1	1
a	1	b	1	0
b	a	1	1	0
1	a	b	1	0

Different Hilbert implications determine the same poset

$\langle A, \rightarrow_{\leq}, 1 \rangle$

$\langle A, \Rightarrow, 1 \rangle$

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a	1	b	1	0
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1	a	b	1	0

$\Rightarrow$	a	b	1	0
0	1	1	1	1
a	1	b	1	b
b	a	1	1	a
1	a	b	1	0

Different Hilbert implications determine the same poset

$\langle A, \to_{\leq}, 1\rangle$

$\to_{\leq}$	a	b	1	0
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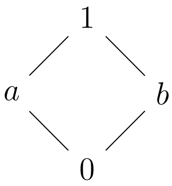
a	1	b	1	0
b	a	1	1	0
1	a	b	1	0

$\langle A, \Rightarrow, 1\rangle$

$\Rightarrow$	a	b	1	0
0	1	1	1	1
a	1	b	1	b
b	a	1	1	a
1	a	b	1	0



Definition (Bounded Distributive Hilbert Algebra)

A *bounded distributive Hilbert algebra*, or DLH-algebra, is an algebra $\langle A, \vee, \wedge, \rightarrow, 0, 1\rangle$ of type $(2, 2, 2, 0, 0)$ such that

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Class of DLH-algebras is denoted by DLHil

Comparison of equations

Heyting algebra

1. $x \to x = 1$

DLH-**algebra**

Comparison of equations

Heyting algebra

1. $x \to x = 1$
2. $y \wedge (x \to y) = y$

DLH-algebra

Comparison of equations

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3. $x \wedge (x \to y) = x \wedge y$

DLH-algebra

Comparison of equations

Heyting algebra

1. $x \to x = 1$
2. $y \wedge (x \to y) = y$
3. $x \wedge (x \to y) = x \wedge y$
4. $x \to (y \wedge z) = (x \to y) \wedge (x \to z)$

DLH-algebra

Comparison of equations

Heyting algebra

1. $x \to x = 1$
2. $y \wedge (x \to y) = y$
3. $x \wedge (x \to y) = x \wedge y$
4. $x \to (y \wedge z) = (x \to y) \wedge (x \to z)$
5. $(x \vee y) \to z = (x \to z) \wedge (y \to z)$

DLH-algebra

Comparison of equations

Heyting algebra

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3. $x \wedge (x \to y) = x \wedge y$
4. $x \to (y \wedge z) = (x \to y) \wedge (x \to z)$
5. $(x \vee y) \to z = (x \to z) \wedge (y \to z)$

DLH-**algebra**

1. $x \wedge (x \to y) = x \wedge y$

Comparison of equations

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1. $x \to x = 1$
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3. $x \wedge (x \to y) = x \wedge y$
4. $x \to (y \wedge z) = (x \to y) \wedge (x \to z)$
5. $(x \vee y) \to z = (x \to z) \wedge (y \to z)$

DLH-algebra

1. $x \wedge (x \to y) = x \wedge y$
2. $(x \to (y \wedge z)) \to ((x \to y) \wedge (x \to z)) = 1$

Comparison of equations

Heyting algebra

1. $x \to x = 1$
2. $y \wedge (x \to y) = y$
3. $x \wedge (x \to y) = x \wedge y$
4. $x \to (y \wedge z) = (x \to y) \wedge (x \to z)$
5. $(x \vee y) \to z = (x \to z) \wedge (y \to z)$

DLH-algebra

1. $x \wedge (x \to y) = x \wedge y$
2. $(x \to (y \wedge z)) \to ((x \to y) \wedge (x \to z)) = 1$
3. $(x \vee y) \to z = (x \to z) \wedge (y \to z)$

Difference

Heyting algebra

$$a \leq b \to c \Leftrightarrow a \wedge b \leq c$$

DLH-**algebra**

$$a \leq b \to c \Rightarrow a \wedge b \leq c$$

Logic	**Algebraic semantic**
Implicative Fragment $\{\to, 1\}$	Hilbert algebra $\langle A, \to, 1\rangle$

Logic	**Algebraic semantic**
Non Intuitionistic logic $\{\vee, \wedge, \rightarrow, 0, 1\}$	DLH algebra $\langle A, \vee, \wedge, \rightarrow, 0, 1\rangle$
Implicative Fragment $\{\rightarrow, 1\}$	Hilbert algebra $\langle A, \rightarrow, 1\rangle$

Example

Let $(A, \vee, \wedge, 0, 1)$ be a bounded distributive lattice and $Fi(A) = \{F \subseteq A : F$ is a lattice filter of $A\}$

$$\langle Fi(A), \cap, \veebar, \Rightarrow, \{1\}, A\rangle \in \mathsf{Hey}$$

$$F \Rightarrow G = \{a \in A : \uparrow a \cap F \subseteq G\}, \quad F, G \in Fi(A)$$

B sublattice of A. Then

$$\langle \mathrm{Fi}(B), \cap, \veebar, \mapsto_B, \{1\}, B\rangle \in \mathsf{DLHil},$$

$$F \mapsto_B H = (\mathrm{Fg}(F) \Rightarrow \mathrm{Fg}(H)) \cap B \quad F, H \in \mathrm{Fi}(B)$$

Definition (Implicative filter)

Let A be a Hilbert algebra. An implicative filter of A $F \in IFi(A)$, is a subset F of A such that

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1. $1 \in F$,
2. If $a, a \to b \in F$, then $b \in F$.

Definition (Irreducible filter)

Let A be a Hilbert algebra. Let F be an implicative filter. Then we say that F is irreducible if and only if

for any $F_1, F_2 \in IFi(A)$ such that $F = F_1 \cap F_2$, then $F = F_1$ or $F = F_2$.

Recall that...

Goal: Representation for DLH-algebras.

The representation in each reduct

$$(A, \vee, \wedge, \to, 0, 1) \in \mathsf{DLHil}$$

$(A, \to, 1)$	$(A, \vee, \wedge, 0, 1)$
$\exists (X, \leq)$	$\exists (X_0, \leq)$
$(A, \to, 1) \hookrightarrow (Up(X), \Rightarrow, X)$	$(A, \vee, \wedge, 0, 1) \hookrightarrow (Up(X_0), \cup, \cap, \emptyset, X)$
$U \Rightarrow V = \{x \in X : [x) \cap U \subseteq V\}$	
$X = Ir(A)$	$X_0 = Pr(A)$

The representation in each reduct

$$(A, \vee, \wedge, \to, 0, 1) \in \mathsf{DLHil}$$

$(A, \to, 1)$	$(A, \vee, \wedge, 0, 1)$
$\beta(a) = \{P \in Ir(A) : a \in P\}$	$\varphi(a) = \{P \in Pr(A) : a \in P\}$
$\beta(a \to b) = \beta(a) \Rightarrow \beta(b)$	$\varphi(a \wedge b) = \varphi(a) \cap \varphi(b)$ $\varphi(a \vee b) = \varphi(a) \cup \varphi(b)$
$\beta(a \wedge b) \neq \beta(a) \cap \beta(b)$	$\varphi(a \to b) \neq \varphi(a) \Rightarrow \varphi(b)$

General true

$$(A, \vee, \wedge, \rightarrow, 0, 1) \in \mathsf{Hey}$$

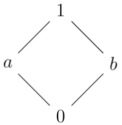
$$Ir(A) = Pr(A)$$

Our setting

$$(A, \vee, \wedge, \rightarrow, 0, 1) \in \mathsf{DLHil}$$

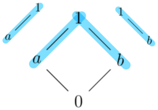
$$Ir(A) \neq Pr(A)$$

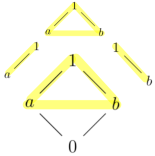
Example



$(A, \wedge, \vee, 0, 1)$ $\qquad$ $(A, \rightarrow_{\leq}, 1)$

$Pr(A) = \{P \in Fi(A) : P \text{ is prime}\}$ $\qquad$ $Ir(A) = \{P \in Fi(A) : P \text{ is irreducible}\}$





$$(A, \wedge, \vee, \to, 0, 1) \in \mathsf{DLHil}$$

$$\boxed{\uparrow Pr(A) = Ir(A)}$$

$$(A, \wedge, \vee, \rightarrow, 0, 1) \in \mathsf{DLHil}$$

$$\boxed{\uparrow Pr(A) = Ir(A)}$$

$$\uparrow \varphi(a) = \beta(a)$$

The representation will be based on:

$$(A, \wedge, \rightarrow, 0, 1)$$

Celani, S. A., & Esteban, M. (2017). Spectral-like duality for distributive Hilbert algebras with infimum. Algebra universalis, 78(2), 193-213.

Definition (Hilbert frame)

A *Hilbert frame* is a structure $\mathcal{F} = \langle Y, \leq, X \rangle$ (or $\mathcal{F} = \langle Y, X \rangle$) such that $\langle Y, \leq \rangle$ is a poset, $X \subseteq Y$, and for every $y \in Y$ there exists $x \in X$ such that $x \leq y$, i.e., $\uparrow X = Y$.

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Proposition

Let $A \in$ DLHil. *Then*

$$\mathcal{F}(A) = \langle \mathrm{Ir}(A), \subseteq, \mathrm{Pr}(A) \rangle$$

is a Hilbert frame, called the canonical Hilbert frame

Recall that

$\langle Y, \leq \rangle$ poset

Recall that

$\langle Y, \leq \rangle$ poset

$$U, V \in Up(Y) \text{ ,then } U \Rightarrow_Y V := \{x \in Y : \uparrow x \cap U \subseteq V\}$$

$\boxed{\langle Y, \leq, X \rangle \text{ Hilbert frame}}$

$U, V \in Up(X)$, then $U \Rightarrow_0 V := (\uparrow U \Rightarrow \uparrow V) \cap X = \{x \in X : \uparrow x \cap \uparrow U \subseteq \uparrow V\}$

$\langle Y, \leq, X \rangle$ Hilbert frame	$\longleftrightarrow$	$A(\mathcal{F}) = \langle Up(X), \cup, \cap, \Rightarrow_0, \emptyset, X \rangle \in$ DLHil

$$U \Rightarrow_0 V := (\uparrow U \Rightarrow \uparrow V) \cap X = \{x \in X : \uparrow x \cap \uparrow U \subseteq \uparrow V\}$$

$\langle Y, \leq, X \rangle$ Hilbert frame	$\longleftrightarrow$	$A(\mathcal{F}) = \langle Up(X), \cup, \cap, \Rightarrow_0, \emptyset, X \rangle \in$ DLHil

$$U \Rightarrow_0 V := (\uparrow U \Rightarrow \uparrow V) \cap X = \{x \in X : \uparrow x \cap \uparrow U \subseteq \uparrow V\}$$

$$\begin{aligned}
\varphi(a \to b) &= \beta(a \to b) \cap Pr(A) \\
&= (\beta(a) \Rightarrow \beta(b)) \cap Pr(A) \\
&= \underbrace{(\uparrow \varphi(a) \Rightarrow \uparrow \varphi(b)) \cap Pr(A)} \\
&=: \qquad \varphi(a) \Rightarrow_0 \varphi(b)
\end{aligned}$$

Theorem ($A \hookrightarrow A(\mathcal{F}(A))$)

Let $A \in$ DLHil. *Then,*

$$\mathcal{F}(A) = \langle \mathrm{Ir}(A), \subseteq, \mathrm{Pr}(A) \rangle$$

is a Hilbert frame, called the canonical frame of A*, and the map*

$$\begin{aligned} \varphi : (A, \vee, \wedge, \rightarrow, 0, 1) &\rightarrow (\mathrm{Up}(Pr(A)), \cup, \cap, \Rightarrow, \emptyset, Pr(A)) \\ a &\mapsto \varphi(a) = \{P \in \mathrm{Pr}(A) : a \in P\} \end{aligned}$$

is an injective homomorphism of distributive Hilbert lattices. Thus, every DLH*-algebra is embeddable into the complex algebra* $A(\mathcal{F}(A))$ *of its canonical frame* $\mathcal{F}(A)$.

Finite DLH

Proposition

Let A be a finite distributive Hilbert algebra. Then $\langle \mathrm{Ir}(A), \mathrm{Pr}(A) \rangle$ is an order-differentiated strong Hilbert frame and the map

$$\varphi : A \to \mathrm{Up}(\mathrm{Pr}(A))$$
$$a \mapsto \varphi(a) = \{P \in \mathrm{Pr}(A) : a \in P\}$$

is a bijective homomorphism of distributive Hilbert algebras.

Finite DLH

Proposition

Let A be a finite distributive Hilbert algebra. Then $\langle \mathrm{Ir}(A), \mathrm{Pr}(A)\rangle$ is an order-differentiated strong Hilbert frame and the map

$$\begin{aligned} \varphi : A &\to \mathrm{Up}(\mathrm{Pr}(A)) \\ a &\mapsto \varphi(a) = \{P \in \mathrm{Pr}(A) : a \in P\} \end{aligned}$$

is a bijective homomorphism of distributive Hilbert algebras.

$$A \cong \mathrm{Up}(\mathrm{Pr}(A))$$

Definition (Hilbert frame order differentiated)

A Hilbert frame $\langle Y, X\rangle$ is called an *order-differentiated*, DiFrHil, if it satisfies that for each $x, y, \in Y$, if $x \nleq y$, then there exists $U \in \mathrm{Up}(X)$ such that $x \in \uparrow U$ and $y \notin \uparrow U$.

Definition (Hilbert frame order differentiated)

A Hilbert frame $\langle Y, X\rangle$ is called an *order-differentiated*, DiFrHil, if it satisfies that for each $x, y, \in Y$, if $x \not\leq y$, then there exists $U \in \mathrm{Up}(X)$ such that $x \in \uparrow U$ and $y \notin \uparrow U$.

Proposition

If A is a DLH*-algebra, then*

$$\mathcal{F}(A) = \langle \mathrm{Ir}(A), \subseteq, \mathrm{Pr}(A)\rangle$$

is an order-differentiated Hilbert frame.

Definition

Let $\langle X_1, Y_1\rangle$ and $\langle X_2, Y_2\rangle$ be two Hilbert frames. A *Hilbert frame morphism* is a map $f : Y_1 \to Y_2$ such that

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Definition

Let $\langle X_1, Y_1\rangle$ and $\langle X_2, Y_2\rangle$ be two Hilbert frames. A *Hilbert frame morphism* is a map $f : Y_1 \to Y_2$ such that

1. f is p-morphism
2. $f\,[X_1] \subseteq X_2$.

Theorem ($\mathcal{F} \hookrightarrow \mathcal{F}(\mathcal{A}(\mathcal{F}))$)

Let $\mathcal{F} = \langle Y, X \rangle$ *be an order-differentiated Hilbert frame. Then the map*

$$\begin{aligned} \varepsilon_Y : \langle Y, X \rangle &\to \langle \mathrm{Ir}(\mathrm{Up}(X)), \mathrm{Pr}(\mathrm{Up}(X)) \rangle \\ y &\mapsto \varepsilon_Y(y) = \{U \in \mathrm{Up}(X) : y \in \uparrow U\} \end{aligned}$$

is an injective Hilbert frame morphism. Thus, every order-differentiated Hilbert frame $\mathcal{F}$ *is embeddable into the canonical frame* $\mathcal{F}(A(\mathcal{F}))$.

The discrete duality provides an equivalence between relational and algebraic semantics.

$\Vdash_{\mathsf{DLHil}} \alpha$ iff $v(\varphi) = 1$,
$v \in Hom(F_m, A), A \in \mathsf{DLHil}$

$\Vdash_{\mathsf{DiFrHil}} \alpha$ iff $v(\varphi) = X$
$v \in Hom(F_m, Up(X)), \langle Y, \leq, X\rangle \in \mathsf{DiFrHil}$

$Th(\mathsf{DLHil}) = \{\alpha : \Vdash_{\mathsf{DLHil}} \alpha\}$

$Th(\mathsf{DiFrHil}) = \{\alpha : \Vdash_{\mathsf{DiFrHil}} \alpha\}$,

$$\mathrm{Th}(\mathsf{DLHil}) = \mathrm{Th}(\mathsf{DiFrHil}),$$

Known result

$$
\begin{tikzcd}
 & \mathsf{Int} \arrow[dl] \arrow[dr] & \\
\mathsf{Hey} \arrow[rr, leftrightarrow] & & \mathsf{Kripke}
\end{tikzcd}
$$

Next goal

$$\begin{tikzcd}
 & ? \arrow[dl] \arrow[dr] & \\
\mathsf{DLH} \arrow[rr, leftrightarrow] & & \mathsf{DiFrHil}
\end{tikzcd}$$

Present and Future work

1. Topological duality for DLH-algebras

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2. Amalgamation properties

Present and Future work

1. Topological duality for DLH-algebras
2. Amalgamation properties
3. Axiomatize the logic associated with DLH-algebras

Bibliography

1. Bezhanishvili, N. (2006). Lattices of intermediate and cylindric modal logics. University of Amsterdam.

Bibliography

1. Bezhanishvili, N. (2006). Lattices of intermediate and cylindric modal logics. University of Amsterdam.
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Bibliography

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3. Celani, S. A., & Esteban, M. (2017). Spectral-like duality for distributive Hilbert algebras with infimum. Algebra universalis, 78(2), 193-213.
4. Celani, S. A., & Montangie, D. (2012). Hilbert algebras with supremum. Algebra universalis, 67(3), 237-255.

Bibliography

1. Bezhanishvili, N. (2006). Lattices of intermediate and cylindric modal logics. University of Amsterdam.
2. Busneag, D., & Ghi, M. (2010). Some latticial properties of Hilbert algebras. Bulletin mathématique de la Société des Sciences Mathématiques de Roumanie, 87-107.
3. Celani, S. A., & Esteban, M. (2017). Spectral-like duality for distributive Hilbert algebras with infimum. Algebra universalis, 78(2), 193-213.
4. Celani, S. A., & Montangie, D. (2012). Hilbert algebras with supremum. Algebra universalis, 67(3), 237-255.
5. Esteban, M. (2013). Duality theory and abstract algebraic logic. Universitat de Barcelona

Bibliography

1. Bezhanishvili, N. (2006). Lattices of intermediate and cylindric modal logics. University of Amsterdam.
2. Busneag, D., & Ghi, M. (2010). Some latticial properties of Hilbert algebras. Bulletin mathématique de la Société des Sciences Mathématiques de Roumanie, 87-107.
3. Celani, S. A., & Esteban, M. (2017). Spectral-like duality for distributive Hilbert algebras with infimum. Algebra universalis, 78(2), 193-213.
4. Celani, S. A., & Montangie, D. (2012). Hilbert algebras with supremum. Algebra universalis, 67(3), 237-255.
5. Esteban, M. (2013). Duality theory and abstract algebraic logic. Universitat de Barcelona
6. Figallo, A. V., Ramón, G., & Saad, S. (1999). A note on the distributive Hilbert algebras. In Proceedings of the Fifth Dr. Antonio AR Monteiro Congress on Mathematics, Bahía Blanca (pp. 139-152).

Bibliography

1. Bezhanishvili, N. (2006). Lattices of intermediate and cylindric modal logics. University of Amsterdam.
2. Busneag, D., & Ghi, M. (2010). Some latticial properties of Hilbert algebras. Bulletin mathématique de la Société des Sciences Mathématiques de Roumanie, 87-107.
3. Celani, S. A., & Esteban, M. (2017). Spectral-like duality for distributive Hilbert algebras with infimum. Algebra universalis, 78(2), 193-213.
4. Celani, S. A., & Montangie, D. (2012). Hilbert algebras with supremum. Algebra universalis, 67(3), 237-255.
5. Esteban, M. (2013). Duality theory and abstract algebraic logic. Universitat de Barcelona
6. Figallo, A. V., Ramón, G., & Saad, S. (1999). A note on the distributive Hilbert algebras. In Proceedings of the Fifth Dr. Antonio AR Monteiro Congress on Mathematics, Bahía Blanca (pp. 139-152).
7. Gehrke, M., & Van Gool, S. (2024). Topological duality for distributive lattices: theory and applications (Vol. 61). Cambridge University Press.

Thank you.