▸ BRANDON WARD, *Failures of choice in the blurry HOD hierarchy.*
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The blurry HOD hierarchy is a weakly increasing hierarchy of inner ZF models introduced by Fuchs [1] that begins with HOD and whose union is the whole universe $V$. For a fixed cardinal $\kappa > 1$, a set belongs to $<\kappa$-HOD if it belongs to an ordinal definable set of size less than $\kappa$, and the same is true for every element in its transitive closure. It is consistent that every level of the hierarchy satisfies choice, and achieving failures of choice at limit levels of the hierarchy is relatively easy. Getting failures of choice at successor levels is harder. The only known case is due to Kanovei and Lyubetsky [2], where they exhibited a forcing extension of $L$ in which $<\omega_1$-HOD fails choice.

In this talk I will present a general framework for achieving failures of choice in $<\kappa^+$-HOD given the existence of what I call a $\kappa$-Kanovei poset. Such posets exist assuming certain $\diamondsuit$ principles and closure conditions on $\kappa$, and allow us to obtain failures of choice in $<\kappa^+$-HOD.

This is joint work with my advisor, Gunter Fuchs.

[1] Gunter Fuchs, *Blurry Definability*, ***Mathematics***, vol. 10 (2022), no. 3, article 452

[2] Vladimir Kanovei, Vassily Lyubetsky, *On Russell typicality in set theory*, ***Proceedings of the American Mathematical Society***, vol. 151 (2023), no. 5, pp. 2201–2210.