

- RAUL FIGUEROA-SIERRA, *The optimal projective complexity of a counterexample to the Category Dichotomy*.

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The Category Dichotomy is a fundamental structural result for Borel ideals on  $\omega$  [3]. The first ZFC counterexample was constructed in [1] via the ideal of nowhere dense subsets of a certain Fréchet space, building on a construction of Dow [2]. However, this work left open whether a counterexample could exist at a low level of the projective hierarchy, such as analytic or co-analytic.

This talk studies this *projective complexity*. We first observe a recent result of O. Guzmán and J. Navarro that the dichotomy holds for analytic ideals. Next, we prove it also holds for the co-analytic class. Our main result establishes the optimality of this boundary: in the constructible universe  $L$ , there exists a  $\Delta_2^1$  ideal constituting a counterexample, thus determining precisely the exact projective level at which the dichotomy can consistently fail.

This is joint work with Osvaldo Guzmán, Michael Hrušák, and Adam Kwela.

[1] A. DOW, R. FIGUEROA-SIERRA, O. GUZMÁN, AND M. HRUŠÁK, *The category dichotomy for ideals*, *Annals of Pure and Applied Logic*, vol. 177 (2026), no. 5, pp. 103717.

[2] A. DOW,  *$\pi$ -weight and the Fréchet-Urysohn property*, *Topology and its Applications*, vol. 174 (2014), pp. 56–61.

[3] MICHAEL HRUŠÁK, *Katětov order on Borel ideals*, *Archive for Mathematical Logic*, vol. 56 (2017), no. 7, pp. 831–847.