▶ MIGUEL AGUILAR, JUAN AGUILERA, AND DAVID FERNÁNDEZ-DUQUE, *The reverse mathematics of the mountain pass theorem.*
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We investigate the metamathematical aspects of the Mountain Pass Theorem (MPT) of Ambrosetti and Rabinowitz [1]. The MPT is a minimax theorem establishing necessary conditions for differentiable functionals to possess critical points and has a wide catalog of applications in PDEs, variational methods, calculus on manifolds, and other domains. Unlike other results of its kind, the critical points provided by MPT are typically saddle points.

We prove that the MPT cannot be established purely on the basis of computable mathematics. More precisely, we prove MPT is equivalent to the non-computable principle WKL_0 over the axiomatic system RCA_0, in the framework of Reverse Mathematics.

Our proof of MPT in WKL_0 deviates from usual proofs and requires establishing computable existence theorems for ODEs on separable Banach spaces, as well as a New Deformation Lemma of independent interest. Doing so isolates precisely where non-computability assumptions are needed for MPT and in turn produces a new version of MPT which is computably true. This new version omits the minimax characterization of saddle points but still guarantees the existence of critical points for functionals.

[1] A. Ambrosetti and P. Rabinowitz, *Dual variational methods in critical point theory and applications*, ***Journal of Functional Analysis***, vol. 14 (1973), no. 4, pp. 349-381.