

- JOÃO MARCOS, *When Attitudes Matter Most: A Bilateral Theory of Consequence*.
Universidade Federal de Santa Catarina, Florianópolis, Brazil.
E-mail: `botocudo@gmail.com`.

The received approach to logical consequence, due to Tarski, takes the judgment of assertion as the single primitive notion underlying inference. On this picture, a logic is a relation between sets of premises and conclusions, which is typically presented by a proof system and determined by classes of logical matrices with a single distinguished set of designated truth-values. This picture, however, is lopsided.

This tutorial explores what happens when a second primitive judgment, namely denial, is placed on an equal footing with assertion. The resulting framework, known as bilateralism, leads naturally to a two-dimensional notion of logical consequence in which a single entailment relation simultaneously tracks the preservation of the cognitive attitude of acceptance and the preservation of the cognitive attitude of rejection. We situate this notion within a spectrum of increasingly general abstract consequence relations, including intermediate non-Tarskian notions that arise naturally between the one-dimensional setting and the richer two-dimensional one.

At the abstract level, the passage to two dimensions resolves a fundamental failure of expressiveness in the one-dimensional theory: the loss of a tight correspondence between consequence relations and their associated spaces of theories. On the semantic side, we develop the theory of two-dimensional non-deterministic matrices, equipped with both designated and anti-designated truth-values. On the proof-theoretic side, we introduce symmetrical Hilbert-style calculi with tree-shaped derivations, which can be effectively extracted from finite sufficiently-expressive matrices and used to perform bounded proof search, with notable consequences for decidability and axiomatizability.

[1] CAROLINA BLASIO, JOÃO MARCOS, AND HEINRICH WANSING, *An inferentially many-valued two-dimensional notion of entailment*, *Bulletin of the Section of Logic*, vol. 46 (2017), no. 3/4, pp. 233–262.

[2] CAROLINA BLASIO, CARLOS CALEIRO, AND JOÃO MARCOS, *What is a logical theory? On theories containing assertions and denials*, *Synthese*, vol. 198 (2021), pp. 5481–5504.

[3] VITOR GREATI, SÉRGIO MARCELINO, AND JOÃO MARCOS, *Proof search on bilateralist judgments over non-deterministic semantics*, *International Conference on Automated Reasoning with Analytic Tableaux and Related Methods* Springer, 2021, pp. 129–146.

[4] VITOR GREATI AND JOÃO MARCOS, *Finite two-dimensional proof systems for non-finitely axiomatizable logics*, *International Joint Conference on Automated Reasoning* Springer, 2022, pp. 640–658.

[5] JOÃO MARCOS, ADAM PŘENOSIL, AND PAUL EGRÉ, *Many-Valued Logic*, *The Stanford Encyclopedia of Philosophy* (Edward N. Zalta and Uri Nodelman, editors), Metaphysics Research Lab, Stanford University, 2026, <https://plato.stanford.edu/entries/logic-manyvalued/>.