

- MARCELO CONIGLIO, *Finite (non-deterministic) truth tables for intuitionistic logic and for modal logics.*

Centre for Logic, Epistemology and the History of Science (CLE), and Institute of Philosophy and the Humanities (IFCH) - State University of Campinas (UNICAMP), Campinas, Brazil.

*E-mail:* coniglio@unicamp.br.

In 1932 Gödel proved that it is impossible to characterize intuitionistic propositional logic (**IPL**) by a single finite logical matrix, that is, by finite-valued truth-tables. By adapting Gödel's proof, J. Dugundji proved in 1940 that no modal system between Lewis's **S1** and **S5** can be characterized by a single finite logical matrix. That is, the usual modal logics are also not characterized by finite-valued truth-tables. As a way to overcome Dugundji's result, J. Kearns [2] introduced in 1981 a 4-valued non-deterministic matrix (Nmatrix, for short) for modal logics **KT**, **S4**, and **S5** in which just a subset of the valuations are allowed (that valuations are called "level valuations"). He proved that this restricted Nmatrix (RNmatrix, for short) constitutes a sound and complete semantics for these modal logics.

Unfortunately, Kearns's level valuations seem to fail to provide an effective decision procedure for any of the modal logics. In 2022, L. Grätz [1] solved this problem for some modal systems. He refined Kearns's original RNmatrix to obtain a decidable 3-valued RNmatrix for modal logics **T** and **S4**. In [4] these results were extended to the full Modal Cube.

Now, recall that in 1933 Gödel proved that there exists a faithful interpretation from **IPL** into **S4**. Since the Gödel translation is computable, then, by composing his algorithm with Grätz's algorithm for **S4**, a decision procedure is immediately obtained for **IPL**. The definition of a *pure* RNmatrix for **IPL** (i.e., without depending on Gödel's translation and Grätz's construction) is a challenge that naturally arises.

After recalling Kearns's approach and Grätz's solution, in this talk, based on [3], we will introduce a sound and complete RNmatrix semantics for **IPL**, as well as a derived decision procedure by means of 3-valued 'truth-tables'. This procedure is obtained by abstracting the composition of Grätz and Gödel's constructions, and the soundness and completeness proofs of our method do not depend on their results. We can conclude that the uncharacterizability problem for **IPL** and modal logics posed by Gödel and Dugundji is overcome by means of the more general notion of 'truth-tables' given by decidable RNmatrices.

[1] L. GRÄTZ, *Truth tables for modal logics T and S4*, **Journal of Logic and Computation**, vol. 32 (2022), no. 1, pp. 129–157.

[2] J. KEARNS, *Modal semantics without possible worlds*, **The Journal of Symbolic Logic**, vol. 46 (1981), no. 1, pp. 77–86.

[3] R. LEME, M.E. CONIGLIO AND B. LOPES, *A new decision method for Intuitionistic Logic by 3-valued non-deterministic truth-tables*, **The Journal of Symbolic Logic**, Accepted manuscript, 2025. DOI: 10.1017/jsl.2025.10174.

[4] R. LEME, C. OLARTE, E. PIMENTEL AND M.E. CONIGLIO, *The Modal Cube Revisited: Semantics without Worlds*, **Automated Reasoning with Analytic Tableaux and Related Methods (TABLEAUX 2025)**, (Reykjavík, Iceland), (G. L. Pozzato and T. Uustalu, editors), LNCS, vol. 15980, Springer Nature, 2026, pp. 181–200.