▶ SYLVY ANSCOMBE, *What can model theory say about power series fields?*.
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Formal power series, with real or complex coefficients, appear throughout mathematics. The set $\mathbb{C}[\![X]\!]$ of all such power series, over $\mathbb{C}$ say, forms a ring under the expected operations, complete with respect to the X-adic topology; moreover it is the valuation ring of the X-adic valuation on its field of fractions $\mathbb{C}(\!(X)\!)$. The first-order theory of $\mathbb{C}(\!(X)\!)$ (in a suitable language of valued fields) was axiomatized, and shown to be decidable, in the 60s by Ax and Kochen, and independently by Ershov: the key ingredient was "henselianity", a first-order relic of completeness. In fact, henselianity together with axioms specifying the theory of the residue field of the valuation also suffices to axiomatize the theory of $F(\!(X)\!)$ for any F of characteristic zero.

In mixed characteristic one analogon is the p-adic field $\mathbb{Q}_p$ which consists of formal sums over powers of p with coefficients from $0, \dots, p-1$. In this setting too, henselianity is the key to unlocking an axiomatization of the complete theory. Similar axiomatizations and decidability results are known for any finitely ramified mixed characteristic henselian valued field.

Things are wildly different in positive characteristic: there is no known axiomatization of the complete theory of $\mathbb{F}_p(\!(t)\!)$, despite it being perhaps the most natural candidate. On the other hand, lots is known by now, first and foremost in the separably tame setting (introduced by Kuhlmann), which generalizes the work of Delon and others on separably algebraically maximal Kaplansky fields. Moreover, the existential portion of the theory of $\mathbb{F}_p(\!(t)\!)$ has been shown to be decidable, by myself and Fehm. On the hypothesis of resolution of singularities, Denef and Schoutens showed that the same is true in a stronger language, allowing the constant t; and wtih Dittmann and Fehm we have shown the same basd on an *a priori* weaker hypothesis. More recent work, again with Fehm, has explored more closely the relationship between weakenings of local uniformization and axiomatizations of fragments of existential theories of power series fields.

I will attempt to survey this topic, including discussion of some enrichments of Hahn series fields by sections and cross-sections.