► LUIZ CARLOS PEREIRA, VICTOR BARROSO-NASCIMENTO AND ELAINE PIMENTEL, *Glivenko's theorems from an ecumenical perspective.*
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In the late twenties and early thirties of last century, several results were obtained concerning some relations between classical/propositional logic (CL/CPL) and intuitionistic/propositional logic (IL/IPL). These interpretations/translations were defined as functions from the language of CL/CPL into some fragment of the language of IL/IPL that aimed to preserve some important properties, like theoremhood or derivability. What is known as *Glivenko's theorems* [2] in the area of logic belongs to this group of important results.
The first Glivenko theorem establishes that if a formula A is classically provable in CPL, then its double negation is intuitionistically provable in IPL.

THEOREM 1. *If* $\vdash_{\mathsf{CPL}} A$*, then* $\vdash_{\mathsf{IPL}} \neg\neg A$.

The second Glivenko theorem establishes that if a formula $\neg A$ is provable CPL, then the same formula is provable in IPL:

THEOREM 2. *If* $\vdash_{\mathsf{CPL}} \neg A$*, then* $\vdash_{\mathsf{IPL}} \neg A$.

In a way, Glivenko's theorems allow classical validities to be sought *constructively*. This allows us to conceive propositional classical logic as a part of propositional intuitionistic logic, the latter being capable of making more *fine-grained* distinctions than the former.

This work examines Glivenko's theorems through the lens of ecumenical logic, focusing on their implications and extensions within a unified logical framework. We begin by revisiting Glivenko's original results and their historical context, emphasizing their significance in bridging the gap between classical and intuitionistic logic. Building on this idea, we explore the application of ecumenical systems, such as those proposed by Prawitz [4], Krauss [3], and Barroso-Nascimento [1], to formalize and generalize Glivenko-type results. Finally, we argue that the ecumenical perspective sheds light on the interplay between classical and intuitionistic *reasoning*, offering a deeper understanding of their coexistence within a single system while respecting their distinct inferential principles.

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[3] Peter Krauss. A constructive interpretation of classical mathematics *Mathematische Schriften Kassel*, preprint No. 5/92, 1992.

[4] Dag Prawitz. Classical versus intuitionistic logic. In Bruno Lopes Edward Hermann Haeusler, Wagner de Campos Sanz, editors, *Why is this a Proof?, Festschrift for Luiz Carlos Pereira*, volume 27, pages 15–32. College Publications, 2015.