

On NIP approximate groups

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Recall that a subset Λ of a group G is said to be an ‘approximate group’ if $1 \in \Lambda = \Lambda^{-1}$ and if the product set $\Lambda^2 = \{ab : a, b \in \Lambda\}$ is covered by finitely many left translates of Λ . In the setting where G and Λ are definable in some structure, the subgroup $\langle \Lambda \rangle$ generated by Λ is naturally an ‘ind-definable’ group in the structure. Groups of this form have been extensively studied from the model-theoretic perspective, among many other places in [3], [7], [5], [4], and [6]. We will discuss work-in-progress investigating the structure of $\langle \Lambda \rangle$ under the assumption that G and Λ are definable in an NIP structure. Among other things, we will give an example where $\langle \Lambda \rangle$ has no type-definable bounded-index subgroup, based on the treatment of $\widetilde{\mathrm{SL}}_2(\mathbb{R})$ in [1], give a new proof of the existence of $\langle \Lambda \rangle^\infty$, originally proved in [2], and discuss connections with Hrushovski’s theorem on ‘generalized locally compact models’ from [4].

References

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