

- JIM DE GROOT¹, JOÃO MARCOS² AND RODRIGO STEFANES², *On the expressive power of modal logics with negative modalities.*

¹ Mathematical Institute, University of Bern, Bern, Switzerland, ² Universidade Federal de Santa Catarina, Florianópolis, Brazil.

E-mail: jim@jimdegroot.com, botocudo@gmail.com, rodrigoamstefanes@gmail.com.

We study the model-theoretic expressive power of modal logics with subclassical negations and restoration modalities. These logics extend positive modal languages with paraconsistent and paracomplete negations, together with additional operators that explicitly recover classical reasoning principles, even though a classical negation is not in general definable. Standard bisimulation is inadequate in this setting, as its symmetry presupposes classical negation.

We introduce a family of simulations parametrized by the modal similarity type, designed to handle negative and restorative modalities while preserving the intensional structure of Kripke models. We prove an adequacy theorem showing that these simulations preserve truth of all formulas in the corresponding restorative modal language.

Using this framework, we establish intrinsic characterization (Hennessy–Milner-type) results for modally saturated Kripke models: modal equivalence coincides with equivalence under the appropriate simulations. From these intrinsic results, we derive relative characterization results (Van Benthem-type), showing that each restorative modal language corresponds exactly to the fragment of first-order logic invariant under its associated simulations.

These results (see details in [1]) place restorative modal logics within the standard landscape of model-theoretic expressivity results for modal logic, extending classical correspondence theory to languages with non-classical negation and explicit restoration operators. They show that such logics form robust, simulation-invariant fragments of first-order logic, despite the absence of a definable classical negation.

[1] JIM DE GROOT, JOÃO MARCOS, RODRIGO STEFANES, *Intrinsic and relative characterization results for logics with negative modalities*, **arXiv:2512.15496**.