Model theory of probability algebras without relative atoms

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In [BH23] the authors gave an axiomatization for the class of atomless probability algebras APA. This class has good model theoretic properties: it is complete, has quantifier elimination, is ω-stable and non forking can be characterized naturally in terms of conditional probabilities. Moreover we have a complete description of the models of APA given by their Maharam's invariants.
Given an algebra $\mathcal{A}$ and a subalgebra $\mathcal{B}$ we can define the notion of relative atom. We say that $a \in \mathcal{A}$ is a relative atom if $a \neq 0$ and for every $c \subseteq a$ we have that $c = a \cap b$ for some $b \in \mathcal{B}$, we say that $\mathcal{A}$ is relatively atomless over $\mathcal{B}$ if for every $a \in \mathcal{A}$ we have that a is not a relative atom. We will extend the language associated to APA by adding a new predicate to the language that defines a subset $\mathcal{B} \subseteq \mathcal{A}$. We define a new theory in the extended language, which we denote as $APAP$, that axiomatizes the class of pairs $(\mathcal{A}, \mathcal{B})$ where $\mathcal{A}$ is relatively atomless over $\mathcal{B}$. We will explain how properties of the theory APA transfer to the theory $APAP$. In addition we will give a description of the models of $APAP$ in terms of their Maharam's invariants for pairs.

Referencias

[BH23] Alexander Berenstein and C. Ward Henson. Model theory of probability spaces. In *Model theory of operator algebras*, volume 11 of *De Gruyter Ser. Log. Appl.*, pages 159–213. De Gruyter, Berlin, 2023.