CONTEXT EVALUATION AS DÉTOUR PROOF-SEARCH FOR PRAWITZIAN GROUNDS

FAVIO E. MIRANDA-PEREA, LOURDES DEL CARMEN GONZÁLEZ-HUESCA, AND EDUARDO UGALDE-REYES

Focusing on normalization results through the Curry–Howard correspondence, we enquire into how proof-theoretic semantics accounts for context evaluation, i.e. the interpretation of evaluation contexts as focusing devices that effect détour proof-search at eliminative occurrences. Building upon the short proofs methodology as presented in [2], and further developed in [3], and extending results of strong normalization for the positive minimal fragment of propositional logic in [4], we develop a typed operational formalization of Dag Prawitz's notion of grounds in which context evaluation is construed as détour proof-search.

Although guided by the Curry–Howard perspective on normalization, we depart from the standard λ- calculus presentation by not taking application as a primitive constructor. Instead, grounds are generated by variables and abstractions together with eliminative structure; evaluation contexts are first-class syntactic objects, and application-like behaviour is recovered by a pair of recursive plugging mechanisms: one inserts an appropriate ground into a basic eliminative scheme (encoding the major premise of a modus ponens whose proof is not yet known), while the other iterates this insertion along a composite elimination scheme.

The operational semantics is organized around β-style reductions that arise only after searching through a context to expose a détour: evaluation propagates a ground through nested elimination schemes until a redex is reached, at which point it is contracted by context substitution. In this way, normalization is internalized as a computational account of Prawitz-style justification rather than as an external metatheorem about derivations. On the proof-theoretic side, we introduce typing judgments for eliminative objects and contexts and prove metatheoretic principles in the style of admissibility and preservation. The result is a proof-theoretic semantics of context evaluation that makes détour proof-search—and hence "direct obtainability" [6]—explicit at the level of computation on grounds.

Keywords: Strong Normalization, Simply Typed Lambda Calculus, Minimal Logic, Curry-Howard correspondence.

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Departamento de Matemáticas, Facultad de Ciencias UNAM, Circuito Exterior S/N, Cd. Universitaria 04510, Coyoacán, Cd.Mx., México.
Email address: favio@ciencias.unam.mx.

Departamento de Matemáticas, Facultad de Ciencias UNAM, Circuito Exterior S/N, Cd. Universitaria 04510, Coyoacán, Cd.Mx., México.
Email address: luglzhuesca@ciencias.unam.mx

Posgrado en Filosofía de la Ciencia UNAM, Unidad de Posgrado, Edificio "E", 1er piso, Circuito de los Posgrados, Ciudad Universitaria, 04510, Coyoacán, Cd. Mx., México.
Email address: edugare@gmail.com