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Bilateralist base-extension semantics with incompatible proofs and refutations.
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The traditional Fregean account of negation depicts denial of a proposition as the mere assertion of its opposite [3]. Logic has traditionally embraced the Fregean view, dealing with rejection only indirectly through definition of assertion conditions for negation. This view has been challenged by contemporary logicians, leading to the creation of what is now known as logical bilateralism [6, 4]. According to bilateralists, assertion and rejection are distinct speech acts that cannot be reduced to each other, so a proper account of logic should distinguish between conditions for propositional assertion and for propositional rejection.

From a proof-theoretic perspective, the distinction between assertion and rejection is important inasmuch it leads to a stronger distinction between *proofs* and *refutations*. Proofs are traditionally conceived as structures capable of guaranteeing epistemic grounds for assertion, and refutations as structures that do the same but for rejection. This claim is reinforced by the observation that development of the intuitionistically acceptable co-implication operator, studied extensively in bi-intuitionistic logics [5, 2], is historically tied to the development of constructive notions of duality [1, 8]. Wansing's bi-intuitionistic system 2Int [7] distinguishes between proof rules and refutation rules, as well as between proof assumptions (or simply assumptions) and refutation assumptions (or contra-assumptions).

In this work, we introduce a new bilateral system, BPR, built upon 2Int, in which the joint derivability of a formula and its refutation does lead to contradiction. We show that BPR enjoys desirable proof-theoretic properties, including normalization. We then define a base-extension semantics for this system, allowing atomic bases to include both atomic proof and atomic refutation rules. We prove that our proposed semantics is sound and complete with respect to the bilateral calculus BPR.

Finally, within this system, we can show that a refutation of A constructively yields a proof of $\neg A$–but not the other way around. This is a noteworthy feature: it reflects the fact that an explicit construction of a counterexample (which our system interprets as a form of refutation) naturally supports the negation of a statement.

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