

- EDWIN D. PATIÑO OSORIO, *Formalization of Lambda Calculus and the Church-Rosser Theorem in Lean4*.

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Lambda calculus is a model of computation that serves as a benchmark for proof assistants. However, standard named notation requires alpha-equivalence to properly define substitution, a process that is complex to implement practically.

To address this, De Bruijn introduced an alternative notation more suitable for machine implementation. This work presents De Bruijn indices [2], representing bound variables as natural numbers based on their abstraction distance. We present a formalization of the Church-Rosser theorem in Lean4, utilizing the Tait-Martin-Löf method via parallel beta-reduction [3]. Furthermore, we discuss the technical challenges in mechanizing the auxiliary lemmas, particularly those concerning index arithmetic and shifting operations [1].

[1] STEFAN BERGHOFER AND CHRISTIAN URBAN, *A Head-to-Head Comparison of de Bruijn Indices and Names*, *Electronic Notes in Theoretical Computer Science*, vol. 174 (2007), no. 5, pp. 53–67.

[2] N. G. DE BRUIJN, *Lambda calculus notation with nameless dummies, a tool for automatic formula manipulation, with application to the Church-Rosser theorem*, *Indagationes Mathematicae (Proceedings)*, vol. 75 (1972), no. 5, pp. 381–392.

[3] M. TAKAHASHI, *Parallel Reductions in λ -Calculus*, *Information and Computation*, vol. 118 (1995), no. 1, pp. 120–127.