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New values of the Julia Robinson number.
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The definability of $\mathbb{N}$ within a ring is a fundamental method to establish the undecidability of its first-order theory. In [1], Julia Robinson established the undecidability for certain rings of totally real algebraic integers. Building upon her results, Vidaux and Videla [3] introduced the Julia Robinson (JR) number as a key invariant for totally real rings to demonstrate such undecidability. We extend the results of Vidaux and Videla concerning the set of JR numbers [4].

By investigating rings defined by quadratic towers $x_n = \sqrt{\nu + \lambda x_{n-1}}$, we provide necessary and sufficient conditions for these towers to be totally real and compute new JR numbers. Depending on the parameters ν, λ, and x_0, we prove these rings satisfy either the JR property (with a JR number of $\lceil \alpha \rceil + \alpha$) or the isolation property (with a value of $\lfloor \alpha \rfloor + \alpha + 1$). This yields new examples of totally real rings with undecidable theories, generalizing previous work by Castillo, Vidaux, and Videla [2].

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[3] X. Vidaux and C. Videla, *Definability of the natural numbers in totally real towers of nested square roots*, ***Proc. Amer. Math. Soc.***, vol. 143 (2015), no. 10, pp. 4463–4477.

[4] C. Muñoz Sandoval, *New values of the Julia Robinson number*, ***CUBO, A Math. J.***, vol. 26 (2024), no. 3, pp. 387–406.