

- VERÓNICA BORJA MACÍAS AND MARCELO E. CONIGLIO AND J. ALEJANDRO HERNANDEZ-TELLO, *Probabilities using LET_K^+* .

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Our work present a study of a non classical probability framework that allow to deal with information with gaps, gluts but also allows to manage the reliability of the information. It is build upon the logic LET_K^+ , a six-valued paracomplete logic that extends the logic *First-Degree Entailment* (FDE). It allows to deal with paraconsistent and paracomplete scenarios but also introduces a recovery operator \circ to restore classical principles like excluded middle and explosion for certain formulas [1].

First we define non-standard twist models $\mathcal{M} = \langle X, v \rangle$ based on a particular twist structure semantics for LET_K^+ . In these models, a valuation v assigns to each formula a triple $(|\phi|^+, |\phi|^-, |\phi|^\circ)$, which is interpreted as sets of states representing positive, negative, and reliable evidence. This triple structure partitions the state space into six distinct regions (e.g., B_φ° reliable belief B_φ belief, D_φ disbelief, D_φ° reliable disbelief, C_φ conflict and U_φ uncertainty for φ in \mathbf{M}) similarly to the four-region proposal using *FDE* in [2].

Building on this semantic foundation, three kind of probability functions are introduced:

1. A one-dimensional function $p(\phi) = \mu(|\phi|^+)$.
2. A three-dimensional "twist" function $\tilde{p}(\phi) = (\mu(|\phi|^+), \mu(|\phi|^-), \mu(|\phi|^\circ))$.
3. A six-dimensional function $\hat{p}(\phi)$ that assigns probabilities directly to the six evidence regions.

There are three central results in the paper, namely the demonstration that these three approaches are equivalent, the soundness and completeness of their syntactic axiomatizations and a generalization of classic update rules but using the LET_K^+ framework. Regarding the last contribution a single-valued Jeffrey update over the partition $\{\phi, \neg\phi\}$ and a more granular six-valued Jeffrey update that operates on the six evidence regions are defined. This finer update allows for belief revision based on detailed changes in the nature of evidence, such as its reliability.

[1] Marcelo E Coniglio and Abilio Rodrigues. From belnap-dunn four-valued logic to six-valued logics of evidence and truth. *Studia Logica*, 112(3):561-606, 2024.

[2] Dominik Klein, Ondrej Majer, and Soroush Rafiee Rad. Probabilities with gaps and gluts. *Journal of Philosophical Logic*, 50(5):1107-1141, 2021.

[3] Brian Weatherson. From Classical to Intuitionistic Probability. *Notre Dame Journal of Formal Logic*, 44(2):111-123, 2003.