▸ WILLIAM ZULUAGA, *A universal algebraic approach to definability in categories.*
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We investigate definability phenomena that emerge when universal algebra meets categorical logic. While definability of operations—via terms or finite families of equations—is a central theme in universal algebra, doctrines capture the categorical essence of logical structure through natural transformations. Our aim is to unite these viewpoints by isolating a purely algebraic fragment of categorical logic where definability can be analyzed with precision.

To this end, we introduce *propositional $\mathcal{V}$-doctrines* (P$\mathcal{V}$Ds): functors $F : \mathsf{C}^{\mathrm{op}} \to \mathcal{V}$, with C possessing finite products and $\mathcal{V}$ a variety of algebras. Finitary F-connectives are natural transformations $F^n \to F$. We develop an equational semantics for P$\mathcal{V}$Ds, based on the class $\mathcal{K}_F = \{F(A) : A \in \mathsf{C}\}$, showing that definability of connectives admits both an internal description in the functor category $\mathcal{V}^{\mathsf{C}^{\mathrm{op}}}$ and an external, universal-algebraic characterization on $\mathcal{K}_F$.

Building on [1], we provide necessary and sufficient conditions for term-definability and for equational definability of F-connectives. We prove that a connective $\eta : F^n \to F$ is term-definable precisely when its canonical value in $F(A^n)$ lies in every projection-generated subalgebra of $F(A)$. For equational definability, we identify the finite systems of equations in $F(A^{n+1})$ that determine η, obtaining preservation criteria for locally finite varieties.

Finally, we analyze representable P$\mathcal{V}$Ds, which extend the classical theory of subobject doctrines studied in the literature. Representability transfers definability questions to the algebraic structure of $F(A)$, yielding a unified and transparent method for classifying internal logical operations in categories equipped with subobject classifiers.

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