▸ MIGUEL ÁNGEL TREJO HUERTA, *Hyperdoctrines over **MLS**: First-Order Domain Theory in Logical Form*.
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We study the category **MLS** of multi-lingual sequent calculus introduced by Kegelmann as a categorical semantics for domain theory in logical form [1]. Objects of **MLS** are consequence relations generated by a sequent calculus with conjunction, disjunction and structural rules but without identity, while morphisms are interpolative consequence relations. Semantically, **MLS** is equivalent to the category of stably compact spaces and closed relations between them.

We observe that the absence of identity in **MLS** yields a natural class of non-classical logics: reflexivity fails, and hence inconsistency does not trivialize and truth is not exhaustive. This makes **MLS** a canonical categorical setting for paraconsistent and paracomplete logics.

Our main contribution is to show how first-order structure can be added to **MLS** by means of hyperdoctrines in the sense of Lawvere. Instead of working over categories of functions, we construct hyperdoctrines over **MLS**, interpreting quantifiers as adjoints to relational reindexing.

We illustrate how standard constructions from categorical logic, such as substitution, quantification and equality-free reasoning, arise naturally in this setting. This provides a unified categorical and proof-theoretic framework for first-order logics without identity, connecting domain theory in logical form, paraconsistency and categorical semantics.

[1] M. Kegelmann, *Continuous Domain Theory in Logical Form, PhD thesis*, ***School of Computer Science, The University of Birmingham***, (1999).

[2] M. A. Moshier, *On the relationship between compact regularity and Gentzen's cut rule*, ***Theoretical Computer Science***, vol. 316 (2004), pp. 113–136.